Blen | Math AA Formula Booklet

Topic 1 Number and Algebra

The nth term of an arithmetic sequence	$u_n = u_1 + (n-1)d$
The sum of n terms of an arithmetic sequence	$S_n = \frac{n}{2}(2u_1 + (n-1)d) = \frac{n}{2}(u_1 + u_n)$
The nth term of a geometric sequence	$u_n = u_1 r^{n-1}$
The sum of n terms of a finite geometric sequence	$S_n = \frac{u_1(r^n - 1)}{r-1} = \frac{u_1(1-r^n)}{r-1},\ r \neq 1$
Compound interest	$FV = PV \times (1 + \frac{r}{100k})^{kn}$,where FV is the future value, PV is the present value, n is the number of years, k is the number of compounding periods per year, $r\%$ is the nominal annual rate of interest
Exponents and logarithms	$a^x = b \Leftrightarrow x = \log_a b$, $where\ a > 0,\ b > 0,\ a \neq 1$
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	$\log_a x^m = m \log_a x$ $\log_a x = \frac{\log_b x}{\log_b a}$
The sum of an infinite geometric sequence	$S_\infty = \frac{u_1}{1-r},\ \lvert r \rvert < 1$
Binomial theorem	$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b^1 + \ldots$ $+ \binom{n}{r} a^{n-r}b^r + \ldots + b^n$ $\binom{n}{r} = nc_r = \frac{n!}{r!\,(n-r)!}$
Combinations	$nc_r = \frac{n!}{r!\,(n-r)!}$
Permutations	$np_r = \frac{n!}{(n-r)!}$
Complex numbers	$z = a + ib$
Modulus-argument (polar) and exponential (Euler) form	$z = r(\cos\theta + i\sin\theta) = re^{i\theta} = r\,\mathrm{cis}\theta$
De Moivre's theorem	$[r(\cos\theta + i\sin\theta)]^n = r^n$ $(\cos n\theta + i\sin n\theta)$ $= r^n e^{in\theta} = r^n \mathrm{cis}\, n\theta$

Prior Learning – SL and HL

Area of a parallelogram	$A = bh$, where b = base, h = height
Area of a triangle	$A = \frac{1}{2}(bh)$, where b = base, h = height
Area of a trapezoid	$A = \frac{1}{2}(a+b)h$, where a, b = parallel sides, h = height
Area of a circle	$A = \pi r^2$, where r = radius
Circumference of a circle	$C = 2\pi r$, where r = radius
Volume of a cuboid	$V = lwh$, where l = length, w = width, h = height
Volume of a cylinder	$V = \pi r^2 h$, where r = radius, h = height
Volume of a prism	$V = Ah$, where A = area of cross-section, h = height
Area of the curved surface of a cylinder	$A = 2\pi rh$, where r = radius, h = height
Distance betwroints (x_1, y_1) and (x_2, y_2)	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
Coordinates of the midpoint of a line segment with endpoints (x_1, y_1) and (x_2, y_2)	$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Topic 2 Functions

Equations of a straight line	$y = mx + c$; $ax + by + d = 0$; $y - y_1 = m(x - x_1)$
Gradient formula	$m = \frac{y_2 - y_1}{x_2 - x_1}$
Axis of symmetry of the graph of a quadratic function	$f(x) = ax^2 + bx + c$ $\Rightarrow$ axis of symmetry is $x = \frac{-b}{2a}$
Solutions of a quadratic equation **Discriminant**	$ax^2 + bx + c = 0$ $\Rightarrow x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a},\ a \neq 0$ $\Delta = b^2 - 4ac$
Exponential and logarithmic functions	$a^x = e^{x\ln a}$; $\log_a a^x = x = a^{\log_a x}$, $where\ a, x > 0,\ a \neq 1$
Sum and product of the roots of polynomial equations of the form $\sum_{r=0}^{n} a_r x^r = 0$	Sum is $\frac{-a_{n-1}}{a_n}$; product is $\frac{(-1)^n a_0}{a_n}$

Topic 3 Geometry and Trigonometry

Distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2)	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$
Coordinates of the midpoint of a line segment with endpoints (x_1, y_1, z_1) and (x_2, y_2, z_2)	$\left(\frac{x_1 - x_2}{2}, \frac{y_1 - y_2}{2}, \frac{z_1 - z_2}{2}\right)$
Volume of a right-pyramid	$V = \frac{1}{3}Ah$, where A = area of the base, h = height
Volume of a right cone	$V = \frac{1}{3}\pi r^2 h$, where r = radius, h = height
Area of the curved surface of a cone	$A = \pi rl$, where r = radius, l = slant height
Volume of a sphere	$V = \frac{4}{3}\pi r^3$, where r = radius
Surface area of a sphere	$A = 4\pi r^2$, where r = radius
Sine rule	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
Cosine rule	$c^2 = a^2 + b^2 - 2ab\cos C$; $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$
Area of a triangle	$A = \frac{1}{2}ab\sin C$
Length of an arc	$l = r\theta$, where r = radius, θ = angle measured in radians
Area of a sector	$A = \frac{1}{2}r^2\theta$, where r = radius, θ = angle measured in radians
Identity for tanθ	$\tan\theta = \frac{\sin\theta}{\cos\theta}$
Pythagorean identity	$\cos^2\theta + \sin^2\theta = 1$
Double angle identities	$\sin 2\theta = 2\sin\theta\cos\theta$ $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

Reciprocal trigo-nometric identi-ties	$\sec\theta = \frac{1}{\cos\theta}$ $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$
Pythagorean identities	$1 + \tan^2\theta = \sec^2\theta$ $1 + \cot^2\theta = \operatorname{cosec}^2\theta$
Compound angle identities Double angle identity for tan	$\sin(A \pm B) =$ $\sin A\cos B \pm \cos A\sin B$ $\cos(A \pm B) =$ $\cos A\cos B \pm \sin A\sin B$ $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \pm \tan A\tan B}$ $\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$
Magnitude of a vector	$\lvert v\rvert = \sqrt{v_1^2 + v_2^2 + v_3^2}$, where $v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$
Scalar product	$v \cdot w = v_1w_1 + v_2w_2 + v_3w_3$, where $v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$, $w = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$ $v \cdot w = \lvert v\rvert\lvert w\rvert\cos\theta$, where θ = angle between v and w
Angle be-tween two vectors	$\cos\theta = \frac{v_1w_1 + v_2w_2 + v_3w_3}{\lvert v\rvert\lvert w\rvert}$
Vector equation of a line	$r = a + \lambda b$
Parametric form of the equation of a line	$x = x_0 + \lambda l, y = y0 + \lambda m, z = z_0 + \lambda n$
Cartesian equations of a line	$\frac{x - x_0}{l} = \frac{y - y_0}{m} = \frac{z - z_0}{n}$
Vector product	$v \times w = \begin{pmatrix} v_2w_3 - v_3w_2 \\ v_3w_1 - v_1w_3 \\ v_1w_2 - v_2w_1 \end{pmatrix}$, where $v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$ $w = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$ $\lvert v \times w\rvert = \lvert v\rvert\lvert w\rvert\sin\theta$, where θ is the angle between v and w
Area of a parallelogram	$A = \lvert v \times w\rvert$ where v and w form two adjacent sides of a parallelogram
Vector equation of a plane	$r = a + \lambda b + \mu c$
Equation of a plane (using the normal vector)	$r \cdot n = a \cdot n$
Cartesian equation of a plane	$ax + by + cz = d$

Topic 4 Statistics and Probability

Interquartile range	$IQR = Q_3 - Q_1$
Mean, x, of a set of data	$\bar{x} = \frac{\sum_{i=1}^{k} f_i x_i}{n}$, where $n = \sum_{i=1}^{k} f_i$
Probability of an event A	$P(A) = \frac{n(A)}{n(U)}$
Complementary events	$P(A) + P(A') = 1$
Combined events	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
Mutually exclusive events	$P(A \cup B) = P(A) + P(B)$
Conditional probability	$P(A\|B) = \frac{P(A \cap B)}{P(B)}$
Independent events	$P(A \cap B) = P(A)\,P(B)$
Expected value of a discrete random variable X	$E(X) = \Sigma x P(X = x)$
Binomial distribution X ~ B (n , p) **Mean** **Variance**	$E(X) = np$ $Var(X) = np(1-p)$
Standardized normal variable	$z = \frac{\chi - \mu}{\sigma}$
Bayes' theorem	$P(B \mid A) = \frac{P(B)\,P(A \mid B)}{P(B)\,P(A \mid B) + P(B')\,P(A \mid B')}$ $P(B_i \mid A) = \frac{P(B_i)\,P(A \mid B_i)}{P(B_1)P(A \mid B_1) + P(B_2)P(A \mid B_2) + P(B_3)P(A \mid B_3)}$
Variance σ²	$\sigma^2 = \frac{\sum_{i=1}^{k} f_i (x_i - \mu)^2}{n} = \frac{\sum_{i=1}^{k} f_i x_i^2}{n} - \mu^2$
Standard deviation σ	$\sigma = \sqrt{\frac{\sum_{i=1}^{k} f_i (x_i - \mu)^2}{n}}$
Linear transformation of a single random variable	$E(aX+b) = aE(X) + b$ $Var(aX+b) = a^2 Var(X)$
Expected value of a continuous random variable X	$E(X) = \mu = \int_{-\infty}^{\infty} x f^2(x)\,dx$
Variance	$Var(X) = E(X-\mu)^2$ $= E(X^2) - [E(X)]^2$
Variance of a discrete random variable X	$Var(X) = \Sigma (x-\mu)^2 P(X = x)\ \Sigma x^2 P(X = x) - \mu^2$
Variance of a continuous random variable X	$Var(X) = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx = \int_{-\infty}^{\infty} x^2 f(x)dx - \mu^2$

Topic 5 Calculus – SL and HL

Derivative of x^n	$f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$
Integral of x^n	$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1$
Area between a curve y = f (x) and the x-axis, where f (x) > 0	$A = \int_a^b y\,dx$
Derivative of sin x	$f(x) = \sin x \Rightarrow f'(x) = \cos x$
Derivative of cos x	$f(x) = \cos x \Rightarrow f'(x) = \sin x$
Derivative of e^x	$f(x) = e^x \Rightarrow f'(x) = e^x$
Derivative of ln x	$f(x) = \ln x \Rightarrow f'(x) = \frac{1}{x}$
Chain rule	$y = g(u)$, where $u = f(x)$ $\Rightarrow \frac{dy}{dx} = \frac{dy}{du} \times \frac{dy}{dx}$
Product rule	$y = uv \Rightarrow u\frac{dv}{dx} + v\frac{du}{dx}$
Quotient rule	$y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{u\frac{dv}{dx} - u\frac{dv}{dx}}{v^2}$
Acceleration	$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$
Distance travelled from t_1 to t_2	$distance = \int_{t2}^{t1} \|v(t)\|\,dt$
Displacement from t_1 to t_2	$displacement = \int_{t1}^{t2} v(t)dt$
Standard integrals	$\int \frac{1}{x}\,dx = \ln\|x\| + C$ $\int \sin x\,dx = -\cos x + C$ $\int \cos dx = -\sin x + C$ $\int e^x\,dx = e^x + C$
Area of region enclosed by a curve and x-axis	$A = \int_a^b \|y\|\,dx$

Topic 5 Calculus – HL only

Derivative of $f(x)$ from first principles	$y = f(x) \Rightarrow \frac{dy}{dx} = f'(x) = \lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$
Standard derivatives	
tan *x*	$f(x) = \tan x \Rightarrow f'(x) = \sec^2 x$
sec *x*	$f(x) = \sec x \Rightarrow f'(x) = \sec x \tan x$
cosec *x*	$f(x) = \operatorname{cosec} x \Rightarrow f'(x) = \operatorname{cosec} x \cot x$
cot *x*	$f(x) = \cot x \Rightarrow f'(x) = \operatorname{cosec}^2 x$
a^x	$f(x) = a^x \Rightarrow f'(x) = a^x(\ln a)$
$\log_a x$	$f(x) = \log_a x \Rightarrow f'(x) = \frac{1}{x \ln a}$
arcsin *x*	$f(x) = \arcsin x \Rightarrow f'(x) = \frac{1}{\sqrt{1-x^2}}$
arccos *x*	$f(x) = \arccos x \Rightarrow f'(x) = \frac{1}{\sqrt{1-x^2}}$
arctan *x*	$f(x) = \arctan x \Rightarrow f'(x) = \frac{1}{1+x^2}$

Standard integrals	$\int a^x\,dx = \frac{1}{\ln a} a^x + C$
	$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C$
	$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \arcsin\left(\frac{x}{a}\right) + C,\ \lvert x\rvert < a$
Integration by parts	$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$ or $\int u\,dv = uv - \int v\,du$
Area of region enclosed by a curve and y-axis	$A = \int_a^b \lvert x\rvert\,dy$
Volume of revolution about the *x* or y-axes	$V = \int_a^b \pi y^2\,dx$ or $V = \int_a^b \pi x^2\,dy$
Euler's method	$y_{n+1} = y_n + h \times f(x_n, y_n)$; $x_{n+1} = x_n + h$, where h is a constant (step length)
Integrating factor for $y' + P(x)y = Q(x)$	$e^{\int P(x)dx}$
Maclaurin series	$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \ldots$
Maclaurin series for special functions	$e^x = 1 + x + \frac{x^2}{2!} + \ldots$
	$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots$
	$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots$
	$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots$
	$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \ldots$

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