Blen | Math AI Formula Booklet

Prior Learning – SL and HL

Area of a parallelogram	$A = bh$, where b = base, h = height
Area of a triangle	$A = \frac{1}{2}(bh)$, where b = base, h = height
Area of a trapezoid	$A = \frac{1}{2}(a+b)h$, where a, b = parallel sides, h = height
Area of a circle	$A = \pi r^2$, where r = radius
Circumference of a circle	$C = 2\pi r$, where r = radius
Volume of a cuboid	$V = lwh$, where l = length, w = width, h = height
Volume of a cylinder	$V = \pi r^2 h$, where r = radius, h = height
Volume of a prism	$V = Ah$, where A = area of cross-section, h = height
Area of the curved surface of a cylinder	$A = 2\pi rh$, where r = radius, h = height
Distance betwroints (x_1,y_1) and (x_2,y_2)	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
Coordinates of the midpoint of a line segment with endpoints (x_1,y_1) and (x_2,y_2)	$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Prior Learning – HL ONLY

Solutions of a quadratic equation	The solutions of $ax^2+bx+c=0$ are $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$, $a \neq 0$

Topic 1 Number and Algebra

The nth term of an arithmetic sequence	$u_n = u_1 + (n-1)d$
The sum of n terms of an arithmetic sequence	$S_n = \frac{n}{2}(2u_1 + (n-1)d) = \frac{n}{2}(u_1 + u_n)$
The nth term of a geometric sequence	$u_n = u_1 r^{n-1}$
The sum of n terms of a finite geometric sequence	$S_n = \frac{u_1(r^n - 1)}{r-1}, \frac{u_1(1-r^n)}{r-1}, r \neq 1$
Compound interest	$FV = PV \times (1+\frac{r}{100k})^{kn}$,where FV is the future value, PV is the present value, n is the number of years, k is the number of compounding periods per year, $r\%$ is the nominal annual rate of interest
Exponents and logarithms	$a^x = b \Leftrightarrow x = \log_a b$, where $a > 0, b > 0, a \neq 1$
Percentage Error	$\varepsilon = \left\lvert\frac{v_A - v_E}{v_E}\right\rvert \times 100\%$, where v_E is the exact value and v_A is the approximate value of v
Laws of logarithms	$\log_a xy = \log_a x + \log_a y$ $\log_a \frac{x}{y} = \log_a x - \log_a y$ $\log_a x^m = m \log_a x$ $for\ a,x,y > 0$
The sum of an infinite geometric sequence	$S_\infty = \frac{u_1}{r-1}, \lvert r \rvert < 1$
Complex numbers	$z = a + ib$
Discriminant	$\Delta = b^2 - 4ac$
Modulus-argument (polar) and exponential (Euler) form	$z = r(\cos\theta + i\sin\theta) = re^{i\theta} = r\,cis\theta$
Determinant of a 2×2 matrix	$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow \det A = \lvert A \rvert = ad - bc$
Inverse of a 2×2 matrix	$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow$ $A^{-1} = \frac{1}{\det A}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}, ad \neq bc$
Power formula for a matrix	$M^n = PD^nP^{-1}$, where P is the matrix of eigenvectors and D is the diagonal matrix of eigenvalues

Topic 2 Functions

Equations of a straight line	$y = mx + c$; $ax + by + d = 0$; $y - y_1 = m(x - x_1)$
Gradient formula	$m = \frac{y_2 - y_1}{x_2 - x_1}$
Axis of symmetry of the graph of a quadratic function	$f(x) = ax^2 + bx + c$ $\Rightarrow$ axis of symmetry is $x = \frac{-b}{2a}$
Logistic function	$f(x) = \frac{L}{1 + Ce^{-kx}}, L, k, C > 0$

Topic 3 Geometry and Trigonometry

Distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2)	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$
Coordinates of the midpoint of a line segment with endpoints (x_1, y_1, z_1) and (x_2, y_2, z_2)	$\left(\frac{x_1 - x_2}{2}, \frac{y_1 - y_2}{2}, \frac{z_1 - z_2}{2}\right)$
Volume of a right-pyramid	$V = \frac{1}{3}Ah$, where A = area of the base, h = height
Volume of a right cone	$V = \frac{1}{3}\pi r^2 h$, where r = radius, h = height
Area of the curved surface of a cone	$A = \pi rl$, where r = radius, l = slant height
Volume of a sphere	$V = \frac{4}{3}\pi r^3$, where r = radius
Surface area of a sphere	$A = 4\pi r^2$, where r = radius
Sine rule	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
Cosine rule	$c^2 = a^2 + b^2 - 2ab\cos C$; $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$
Area of a triangle	$A = \frac{1}{2}ab\sin C$
Length of an arc	$l = \frac{\theta}{360} \times 2\pi r$, where r = radius, θ = angle measured in degrees
Area of a sector	$A = \frac{\theta}{360} \times \pi r^2$, where r = radius, θ = angle measured in degrees

Length of an arc	$l = r\theta$, where r=radius, θ=angle measured in radians
Area of a sector	$A = \frac{1}{2}r^2\theta$, where r=radius, θ=angle measured in radians
Identities	$\cos^2\theta + \sin^2\theta = 1$; $\tan\theta = \frac{\sin\theta}{\cos\theta}$
Transformation matrices	$\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$, reflection in the line $y = (\tan\theta)x$ $\begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix}$, horizontal stretch/ stretch parallel to x- axis with a scale factor of k $\begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix}$, vertical stretch/ stretch parallel to y- axis with a scale factor of k $\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$, enlargement with a scale factor of k, centre $(0,0)$ $\begin{pmatrix} \cos 2\theta & -\sin\theta \\ \sin 2\theta & \cos\theta \end{pmatrix}$, anticlockwise/ counterclockwise rotation of angle θ about the origin $(\theta > 0)$ $\begin{pmatrix} \cos 2\theta & \sin\theta \\ -\sin 2\theta & \cos\theta \end{pmatrix}$, clockwise rotation of angle θ about the origin $(\theta > 0)$
Magnitude of a vector	$\lvert v \rvert = \sqrt{v_1^2 + v_2^2 + v_3^2}$, where $v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$
Vector equation of a line	$r = a + \lambda b$
Parametric form of the equation of a line	$x = x_0 + \lambda l, y = y_0 + \lambda m, z = z_0 + \lambda n$
Scalar product	$v \cdot w = v_1 w_1 + v_2 w_2 + v_3 w_3$, where $v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$, $w = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$ $v \cdot w = \lvert v \rvert \lvert w \rvert \cos\theta$, where θ = angle between v and w
Angle be-tween two vectors	$\cos\theta = \frac{v_1 w_1 + v_2 w_2 + v_3 w_3}{\lvert v \rvert \lvert w \rvert}$
Vector product	$v \times w = \begin{pmatrix} v_2 w_3 - v_3 w_2 \\ v_3 w_1 - v_1 w_3 \\ v_1 w_2 - v_2 w_1 \end{pmatrix}$, where $v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$ $w = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$ $\lvert v \times w \rvert = \lvert v \rvert \lvert w \rvert \sin\theta$, where θ is the angle between v and w
Area of a parallelogram	$A = \lvert v \times w \rvert$ where v and w form two adjacent sides of a parallelogram

Topic 4 Statistics and probability

Interquartile range	$IQR = Q_3 - Q_1$
Mean, $\bar{x}$, of a set of data	$\bar{x} = \frac{\sum_{i=1}^{k} f_i x_i}{n}$, where $n = \sum_{i=1}^{k} f_i$
Probability of an event A	$P(A) = \frac{n(A)}{n(U)}$
Complementary events	$P(A) + P(A') = 1$
Combined events	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
Mutually exclusive events	$P(A \cup B) = P(A) + P(B)$
Conditional probability	$P(A\|B) = \frac{P(A \cap B)}{P(B)}$
Independent events	$P(A \cap B) = P(A)\, P(B)$
Expected value of a discrete random variable X	$E(X) = \Sigma x P(X = x)$
Binomial distribution $X \sim B(n, p)$ **Mean** **Variance**	$E(X) = np$ $Var(X) = np(1 - p)$
Linear transformation of a single random variable	$E(aX + b) = aE(X) + b$ $Var(aX + b) = a^2 Var(X)$
Linear combinations of n independent random variables, $X_1, X_2, \ldots X_3$	$E(a_1X_1 \pm a_2X_2 \pm \ldots \pm a_nX_n)$ $= a_1E(X_1) \pm a_2E(X_2) \pm \ldots \pm a_nE(X_n)$ $Var(a_1X_1 \pm a_2X_2 \pm \ldots \pm a_nX_n)$ $= a_1^2 Var(X_1) + a_2^2 Var(X_2) + \ldots +$ $a_n^2 Var(X_n)$
Sample statistics **Unbiased estimate of population variance S^2_{n-1}**	$s^2_{n-1} = \frac{n}{n - 1} = s^2_n$
Poisson distribution $X \sim Po(m)$ **Mean** **Variance**	$E(X) = m$ $Var(X) = m$
Transition matrices	$T^n s_0 = s_n$, where s_0 is the initial state

Topic 5 Calculus

Derivative of x^n	$f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$
Integral of x^n **Area between a curve $y = f(x)$ and the x-axis, where $f(x) > 0$**	$\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$ $A = \int_a^b y\, dx$
The trapezoidal rule	$\int_a^b y dx \approx \frac{1}{2} h\{ (y_0 + y_n) +$ $2(y_1 + y_2 + \ldots + y_{n-1})\}$ Where $h = \frac{b-a}{n}$
Derivative of sin x	$f(x) = \sin x \Rightarrow f'(x) = \cos x$
Derivative of cos x	$f(x) = \cos x \Rightarrow f'(x) = -\sin x$
Derivative of e^x	$f(x) = e^x \Rightarrow f'(x) = e^x$
Derivative of ln x	$f(x) = \ln x \Rightarrow f'(x) = \frac{1}{x}$
Chain rule	$y = g(u)$, where $u = f(x) \Rightarrow$ $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
Product rule	$y = uv \Rightarrow u\frac{dv}{dx} + \frac{du}{dx}$
Quotient rule	$y = \frac{u}{v} \Rightarrow \frac{dv}{dx} = \frac{u\frac{dv}{dx} + \frac{du}{dx}}{v^2}$
Standard integrals	$\int \frac{1}{x} dx = \ln\|x\| + C$ $\int \sin dx = -\cos x + C$ $\int \cos dx = -\sin x + C$ $\frac{1}{\cos 2x} = \tan x + C$ $\int e^x dx = e^x + C$
Area of region enclosed by a curve and x or y-axes	$A = \int_a^b \|y\|\, dx$ or $A = \int_a^b \|x\|\, dy$
Volume of revolution about the x or y-axes	$V = \int_a^b \pi y^2\, dx$ or $V = \int_a^b \pi x^2\, dy$
Acceleration	$a \Rightarrow \frac{dv}{dt} = \frac{d^2s}{dt^2} = v\frac{dv}{ds}$
Distance travelled from t_1 to t_2	$distance = \int_{t_1}^{t_2} \|v(t)\|\, dt$
Displacement from t_1 to t_2	$displacement = \int_{t_1}^{t_2} v(t) dt$
Euler's method	$y_{n+1} = y_n + h \times f(x_n, y_n); x_{n+1} = x_n +$ h, where h is a constant (step length)
Euler's method for coupled systems	$x_{n+1} = x_n + h \times f(x_n, y_n, t_n);$ $y_{n+1} = y_n + h \times f(x_n, y_n, t_n);$ $t_{n+1} = t_n + h$ where h is a constant (step length)
Exact solution for coupled linear differential equations	$x = Ae^{\lambda_1 t} p_1 + Be^{\lambda_2 t} p_2$

Adapted from IB Mathematics Application and Interpretation Formula Booklet, published in 2019