

Mathematical equations

Area of a circle	$A = \pi r^2$, where r is the radius
Circumference of a circle	$C = 2\pi r$, where r is the radius
Surface area of a sphere	$A = 4\pi r^2$, where r is the radius
Volume of a sphere	$V = \frac{4}{3}\pi r^3$, where r is the radius

Fundamental constants

Quantity	Symbol	Approximate value
Acceleration of free fall (Earth's surface)	g	9.81ms ⁻²
Gravitational constant	G	6.67 x 10 ⁻¹¹ Nm ² kg ⁻²
Avogadro's constant	N_A	6.02 x 10 ²³ mol ⁻¹
Gas constant	R	8.31JK ⁻¹ mol ⁻¹
Boltzmann's constant	k_B	1.38 x 10 ⁻²³ JK ⁻¹
Stefan-Boltzmann constant	σ	5.67 x 10 ⁻⁸ Wm ² K ⁻⁴
Coulomb constant	k	8.99 x 10 ⁹ Nm ² C ⁻²
Permittivity of free space	ϵ_0	8.85 x 10 ⁻¹² C ² N ⁻¹ m ⁻²
Permeability of free space	μ_0	4π x 10 ⁻⁷ TmA ⁻¹
Speed of light in vacuum	c	3.00 x 10 ⁸ ms ⁻¹
Planck's constant	h	6.63 x 10 ⁻³⁴ Js
Elementary charge	e	1.60 x 10 ⁻¹⁹ C
Electron rest mass	m_e	9.110 x 10 ⁻³¹ kg = 0.000549u = 0.511MeVc ⁻²
Proton rest mass	m_p	1.673 x 10 ⁻²⁷ kg = 1.007276u = 938MeVc ⁻²
Neutron rest mass	m_n	1.675 x 10 ⁻²⁷ kg = 1.008665u = 940MeVc ⁻²
Unified atomic mass unit	u	1.661 x 10 ⁻²⁷ kg = 931.5MeVc ⁻²
Solar constant	S	1.36 x 10 ³ Wm ⁻²
Fermi radius	R_0	1.20 x 10 ⁻¹⁵ m

Unit conversions

1 radian (rad) = $\frac{180^\circ}{\pi}$
Temperature (K) = temperature (°C) + 273
1 light year (ly) = 9.46 x 10 ¹⁵ m
1 parsec (pc) = 3.26ly
1 astronomical unit (AU) = 1.50 x 10 ¹¹ m
1kilowatt-hour (kWh) = 3.60 x 10 ⁶ J
hc = 1.99 x 10 ⁻²⁵ Jm = 1.24 x 10 ⁻⁶ eVm

Metric (SI) multipliers

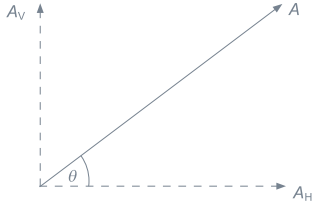
Prefix	Abbreviation	Value
peta	P	10 ¹⁵
tera	T	10 ¹²
giga	G	10 ⁹
mega	M	10 ⁶
kilo	k	10 ³
hecto	h	10 ²
deca	da	10 ¹
deci	d	10 ⁻¹
centi	c	10 ⁻²
milli	m	10 ⁻³
micro	μ	10 ⁻⁶
nano	n	10 ⁻⁹
pico	p	10 ⁻¹²
femto	f	10 ⁻¹⁵

Electrical circuit symbols

cell		battery	
ac supply		switch	
voltmeter		ammeter	
resistor		variable resistor	
lamp		potentiometer	
light-dependent resistor (LDR)		thermistor	
transformer		heating element	
diode		capacitor	

Equations—Core

Note: All equations relate to the magnitude of the quantities only. Vector notation has not been used.

1.2 Uncertainties and errors	1.3 Vectors and scalars
<p>If: $y = a \pm b$ then: $\Delta y = \Delta a + \Delta b$</p> <p>If: $y = \frac{ab}{c}$ then: $\frac{\Delta y}{y} = \frac{\Delta a}{a} + \frac{\Delta b}{b} + \frac{\Delta c}{c}$</p> <p>If: $y = a^n$ then: $\frac{\Delta y}{y} = \left n \frac{\Delta a}{a} \right$</p>	 <p style="text-align: center;">$A_H = A \cos \theta$ $A_V = A \sin \theta$</p>
2.1 Motion	2.2 Forces
<p>$v = u + at$ $s = ut + \frac{1}{2}at^2$ $v^2 = u^2 + 2as$ $s = \frac{(v+u)t}{2}$</p>	<p>$F = ma$ $F_f \leq \mu_s R$ $F_f = \mu_d R$</p>
2.3 Work, energy and power	2.4 Momentum and impulse
<p>$W = Fs \cos \theta$ $E_k = \frac{1}{2}mv^2$ $E_p = \frac{1}{2}k\Delta x^2$ $\Delta E_p = mg\Delta h$ power = Fv efficiency = $\frac{\text{useful work out}}{\text{total work in}}$ $= \frac{\text{useful power out}}{\text{total power in}}$</p>	<p>$p = mv$ $F = \frac{\Delta p}{\Delta t}$ $E_k = \frac{p^2}{2m}$ impulse = $F\Delta t = \Delta p$</p>
3.1 Thermal concepts	3.2 Modelling a gas
<p>$Q = mc\Delta T$ $Q = mL$</p>	<p>$p = \frac{F}{A}$ $n = \frac{N}{N_A}$ $pV = nRT$ $\bar{E}_k = \frac{3}{2}k_B T = \frac{3}{2} \frac{R}{N_A} T$</p>
4.1 Oscillations	4.4 Wave behaviour
<p>$T = \frac{1}{f}$</p>	<p>$\frac{n_2}{n_1} = \frac{\sin \theta_2}{\sin \theta_1} = \frac{v_2}{v_1}$ $s = \frac{\lambda D}{d}$</p> <p>Constructive interference: path difference = $n\lambda$</p> <p>Destructive interference: path difference = $(n + \frac{1}{2})\lambda$</p>
4.2 Travelling waves	
<p>$c = f\lambda$</p>	
4.3 Wave characteristics	
<p>$I \propto A^2$ $I \propto x^{-2}$ $I = I_0 \cos^2 \theta$</p>	

5.1 Electric fields	5.2 Heating effect of electric currents																											
<p>$I = \frac{\Delta q}{\Delta t}$ $F = k \frac{q_1 q_2}{r^2}$ $k = \frac{1}{4\pi\epsilon_0}$ $V = \frac{W}{q}$ $E = \frac{F}{q}$ $I = nAvq$</p>	<p>Kirchhoff's circuit laws: $\Sigma V = 0$ (loop) $\Sigma I = 0$ (junction)</p> <p>$R = \frac{V}{I}$ $P = VI = I^2 R = \frac{V^2}{R}$ $R_{total} = R_1 + R_2 + \dots$ $\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots$ $\rho = \frac{RA}{L}$</p>																											
5.3 Electric cells	5.4 Magnetic effects of electric currents																											
<p>$\epsilon = I(R+r)$</p>	<p>$F = qvB \sin \theta$ $F = BIL \sin \theta$</p>																											
6.1 Circular motion	6.2 Newton's law of gravitation																											
<p>$v = \omega r$ $a = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$ $F = \frac{mv^2}{r} = m\omega^2 r$</p>	<p>$F = G \frac{Mm}{r^2}$ $g = \frac{F}{m}$ $g = G \frac{M}{r^2}$</p>																											
7.1 Discrete energy and radioactivity	7.2 Nuclear reactions																											
<p>$E = hf$ $\lambda = \frac{hc}{E}$</p>	<p>$\Delta E = \Delta mc^2$</p>																											
7.3 The structure of matter																												
<table border="1" style="width: 100%;"> <thead> <tr> <th>Charge</th> <th colspan="3">Quarks</th> <th>Baryon number</th> <th>Charge</th> <th colspan="3">Leptons</th> </tr> </thead> <tbody> <tr> <td>$\frac{2}{3}e$</td> <td>u</td> <td>c</td> <td>t</td> <td>$\frac{1}{3}$</td> <td>-1</td> <td>e</td> <td>μ</td> <td>τ</td> </tr> <tr> <td>$-\frac{1}{3}e$</td> <td>d</td> <td>s</td> <td>b</td> <td>$\frac{1}{3}$</td> <td>0</td> <td>ν_e</td> <td>ν_μ</td> <td>ν_τ</td> </tr> </tbody> </table> <p>All quarks have a strangeness number of 0 except the strange quark that has a strangeness number of -1</p>	Charge	Quarks			Baryon number	Charge	Leptons			$\frac{2}{3}e$	u	c	t	$\frac{1}{3}$	-1	e	μ	τ	$-\frac{1}{3}e$	d	s	b	$\frac{1}{3}$	0	ν_e	ν_μ	ν_τ	<p>All leptons have a lepton number of 1 and antileptons have a lepton number of -1</p>
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8.1 Energy sources	8.2 Thermal energy transfer																											
<p>power = $\frac{\text{energy}}{\text{time}}$ power = $\frac{1}{2}Aqv^3$</p>	<p>$P = \epsilon\sigma AT^4$ $\lambda_{max} (\text{metres}) = \frac{2.90 \times 10^{-3}}{T (\text{kelvin})}$ $I = \frac{\text{power}}{A}$ albedo = $\frac{\text{total scattered power}}{\text{total incident power}}$</p>																											

Equations—AHL

<p>9.1 Simple harmonic motion</p> $\omega = \frac{2\pi}{T}$ $a = -\omega^2 x$ $x = x_0 \sin \omega t; x = x_0 \cos \omega t$ $v = \omega x_0 \cos \omega t; v = \omega x_0 \sin \omega t$ $v = \pm \omega \sqrt{(x_0^2 - x^2)}$ $E_k = \frac{1}{2} m \omega^2 (x_0^2 - x^2)$ $E_T = \frac{1}{2} m \omega^2 x_0^2$ <p>pendulum: $T = 2\pi \sqrt{\frac{l}{g}}$</p> <p>mass-spring: $T = 2\pi \sqrt{\frac{m}{k}}$</p>	<p>9.2 Single-slit diffraction</p> $\theta = \frac{\lambda}{b}$ <p>9.3 Interference</p> $n\lambda = d \sin \theta$ <p>Constructive interference: $2dn = m\lambda$</p> $= \left(m + \frac{1}{2}\right)\lambda$ <p>Destructive interference: $2dn = m\lambda$</p>	<p>11.1 Electromagnetic induction</p> $\Phi = BA \cos \theta$ $\varepsilon = -N \frac{\Delta \Phi}{\Delta t}$ $\varepsilon = Bvl$ $\varepsilon = BvLN$ <p>11.2 Power generation and transmission</p> $I_{rms} = \frac{I_o}{\sqrt{2}}$ $V_{rms} = \frac{V_o}{\sqrt{2}}$ $R = \frac{V_o}{I_o} = \frac{V_{rms}}{I_{rms}}$ $P_{max} = I_o V_o$ $\bar{P} = \frac{1}{2} I_o V_o$ $\frac{\varepsilon_p}{\varepsilon_s} = \frac{N_p}{N_s} = \frac{I_p}{I_s}$	<p>11.3 Capacitance</p> $C = \frac{q}{V}$ $C_{parallel} = C_1 + C_2 + \dots$ $\frac{1}{C_{series}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$ $C = \varepsilon \frac{A}{d}$ $E = \frac{1}{2} CV^2$ $\tau = RC$ $q = q_0 e^{-t/\tau}$ $I = I_0 e^{-t/\tau}$ $V = V_0 e^{-t/\tau}$								
<p>9.4 Resolution</p> $\theta = 1.22 \frac{\lambda}{b}$ $R = \frac{\lambda}{\Delta \lambda} = mN$	<p>9.5 Doppler effect</p> <p>Moving source: $f' = f \left(\frac{v}{v \pm u_s} \right)$</p> <p>Moving observer: $f' = f \left(\frac{v \pm u_o}{v} \right)$</p> $\frac{\Delta f}{f} = \frac{\Delta \lambda}{\lambda} \approx \frac{v}{c}$	<p>12.1 The interaction of matter with radiation</p> $E = hf$ $E_{max} = hf - \Phi$ $E = \frac{13.6}{n^2} eV$ $mvr = \frac{nh}{2\pi}$ $Pr(r) = \psi ^2 \Delta V$ $\Delta x \Delta p \geq \frac{h}{4\pi}$ $\Delta E \Delta t \geq \frac{h}{4\pi}$	<p>12.2 Nuclear physics</p> $R = R_0 A^{\frac{1}{3}}$ $N = N_0 e^{-\lambda t}$ $A = \lambda N_0 e^{-\lambda t}$ $\sin \theta \approx \frac{\lambda}{D}$								
<p>10.1 Describing fields</p> $W = q\Delta V_e$ $W = m\Delta V_g$	<p>10.2 Fields at work</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="padding: 5px;">$V_g = -\frac{GM}{r}$</td> <td style="padding: 5px;">$V_e = -\frac{kQ}{r}$</td> </tr> <tr> <td style="padding: 5px;">$g = -\frac{\Delta V_g}{\Delta r}$</td> <td style="padding: 5px;">$E = -\frac{\Delta V_e}{\Delta r}$</td> </tr> <tr> <td style="padding: 5px;">$E_p = mV_g$ $= -\frac{GMm}{r}$</td> <td style="padding: 5px;">$E_p = qV_e$ $= \frac{kQq}{r}$</td> </tr> <tr> <td style="padding: 5px;">$F_g = \frac{GMm}{r^2}$</td> <td style="padding: 5px;">$F_e = \frac{kQq}{r^2}$</td> </tr> </table> $V_{esc} = \sqrt{\frac{2GM}{r}}$ $V_{orbit} = \sqrt{\frac{GM}{r}}$	$V_g = -\frac{GM}{r}$	$V_e = -\frac{kQ}{r}$	$g = -\frac{\Delta V_g}{\Delta r}$	$E = -\frac{\Delta V_e}{\Delta r}$	$E_p = mV_g$ $= -\frac{GMm}{r}$	$E_p = qV_e$ $= \frac{kQq}{r}$	$F_g = \frac{GMm}{r^2}$	$F_e = \frac{kQq}{r^2}$		
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Equations—Options

A.1 The beginnings of relativity	A.2 Lorentz transformations
$x' = x - vt$ $u' = u - v$	$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$ $x' = \gamma(x - vt); \Delta x' = \gamma(\Delta x - v\Delta t)$ $t' = \gamma\left(t - \frac{vx}{c^2}\right); \Delta t' = \gamma\left(\Delta t - \frac{v\Delta x}{c^2}\right)$ $u' = \frac{u - v}{1 - \frac{uv}{c^2}}$ $\Delta t = \gamma\Delta t_0$ $L = \frac{L_0}{\gamma}$ $(ct')^2 - (x')^2 = (ct)^2 - (x)^2$
A.3 Spacetime diagrams	
$\theta = \tan^{-1}\left(\frac{v}{c}\right)$	
A.4 Relativistic mechanics (HL only)	A.5 General relativity (HL only)
$E = \gamma m_0 c^2$ $E_0 = m_0 c^2$ $E_k = (\gamma - 1)m_0 c^2$ $p = \gamma m_0 v$ $E^2 = p^2 c^2 + m_0^2 c^4$ $qV = \Delta E_k$	$\frac{\Delta f}{f} = \frac{g\Delta h}{c^2}$ $R_s = \frac{2GM}{c^2}$ $\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{R_s}{r}}}$

B.3 Fluids and fluid dynamics (HL only)	B.4 Forced vibrations and resonance (HL only)
$B = \rho_f V_f g$ $P = P_0 + \rho_f g d$ $Av = \text{constant}$ $\frac{1}{2}\rho v^2 + \rho g z + p = \text{constant}$ $F_D = 6\pi\eta r v$ $R = \frac{vr_0}{\eta}$	$Q = 2\pi \frac{\text{energy stored}}{\text{energy dissipated per cycle}}$ $Q = 2\pi \times \text{resonant frequency}$ $\times \frac{\text{energy stored}}{\text{power loss}}$

C.1 Introduction to imaging	C.2 Imaging instrumentation
$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$ $P = \frac{1}{f}$ $m = \frac{h_i}{h_o} = -\frac{v}{u}$ $M = \frac{\theta_i}{\theta_o}$ $M_{\text{near point}} = \frac{D}{f} + 1; M_{\text{infinity}} = \frac{D}{f}$	$M = \frac{f_p}{f_c}$
	C.3 Fibre optics
	$n = \frac{c}{\sin c}$ $\text{attenuation} = 10 \log \frac{I}{I_0}$
	C.4 Medical imaging (HL only)
	$L_i = 10 \log \frac{I_i}{I_0}$ $I = I_0 e^{-\mu x}$ $\mu x_{\frac{1}{2}} = \ln 2$ $Z = \rho c$

B.1 Rigid bodies and rotational dynamics	B.2 Thermodynamics
$\Gamma = Fr \sin \theta$ $I = \sum m r^2$ $\Gamma = I\alpha$ $\omega = 2\pi f$ $\omega_f = \omega_i + \alpha t$ $\omega_f^2 = \omega_i^2 + 2\alpha\theta$ $\theta = \omega_f t + \frac{1}{2}\alpha t^2$ $L = I\omega$ $E_{K_{rot}} = \frac{1}{2}I\omega^2$	$Q = \Delta U + W$ $U = \frac{3}{2}nRT$ $\Delta S = \frac{\Delta Q}{T}$ $pV^{\frac{5}{3}} = \text{constant}$ <p>(for monatomic gases)</p> $W = p\Delta V$ $\eta = \frac{\text{useful work done}}{\text{energy input}}$ $\eta_{\text{Carnot}} = 1 - \frac{T_{\text{cold}}}{T_{\text{hot}}}$

D.1 Stellar quantities	D.2 Stellar characteristics and stellar evolution
$d \text{ (parsec)} = \frac{1}{p \text{ (arc-second)}}$ $L = \alpha AT^4$ $b = \frac{L}{4\pi d^2}$	$\lambda_{\text{max}} T = 2.9 \times 10^{-3} \text{ mK}$ $L \propto M^{3.5}$
D.3 Cosmology	D.5 Further cosmology (HL only)
$z = \frac{\Delta\lambda}{\lambda_0} \approx \frac{v}{c}$ $z = \frac{R}{R_0} - 1$ $v = H_0 d$ $T \approx \frac{1}{H_0}$	$v = \sqrt{\frac{4\pi G \rho}{3}} r$ $\rho_c = \frac{3H^2}{8\pi G}$